Appendix

Consider formula (6) in the vicinity of equiprobable point

$$p_i = \frac{1}{N} + \varepsilon_i, i = 1, 2, \ldots, N, \sum_i \varepsilon_i = 0$$

Since

$$\left(p_{i_1} + p_{i_2} + \ldots + p_{i_\nu} \right) = \frac{\nu}{N} + (\varepsilon_{i_1} + \varepsilon_{i_2} + \ldots + \varepsilon_{i_\nu})$$

Then

$$E_\upsilon = \frac{\upsilon}{N} + \frac{\sum_{P(\upsilon)} (\varepsilon_{i_1} + \varepsilon_{i_2} + \ldots + \varepsilon_{i_\upsilon}) T\left(P(\upsilon)\right)}{\sum_{P(\upsilon)} T\left(P(\upsilon)\right)} \qquad (1A)$$

Expression

$$\tilde{E}_\upsilon = \frac{\sum_{P(\upsilon)} (\varepsilon_{i_1} + \varepsilon_{i_2} + \ldots + \varepsilon_{i_\upsilon}) T\left(P(\upsilon)\right)}{\sum_{P(\upsilon)} T\left(P(\upsilon)\right)} \qquad (2A)$$

(the second term in (1A)), is a function of variables ε_*. Calculating $\tilde{E}_\upsilon$ with the second-order accuracy with respect to ε_*, note that the numerator and the denominator in (2A) are symmetric polynomials in ε and, therefore, can be represented as polynomials in the elementary symmetric polynomials. However, the elementary first-order symmetric polynomial in ε_* is identical to zero by the definition of ε_*, so that the numerator in (2A) contains only the quadratic term in the required approximation, which, obviously, originates the linear term in expression. In this case, in the denominator of (2A), it is only necessary to find the term independent of ε_*. Thus in expression $T(P(v))$, the independent of ε_* and the linear in it terms must be calculated.

For the sake of the description convenience, denote

$$C_1 = (1 - p_{i_1} - p_{i_2} - \ldots - p_{i_\nu})^n \qquad (3A)$$

$$C_2 = (1 - \sum_j^* \left(1 - \frac{p_j}{\omega}\right)^n - \sum^*_{j_t < j_s} (1 - \frac{p_{j\,t}}{\omega} - \frac{p_{j_s}}{\omega})^n + \ldots)) \qquad (4A)$$

Then

$$T(P(v)) = C_1 C_2 \qquad (5A)$$

In linear approximation,

$$C_1 = (1 - \frac{v}{N})^n - n(1 - \frac{v}{N})^{n-1} (\sum_{P(v)} \varepsilon_i) \qquad (6A)$$

Consider now

$$\frac{p_j}{\omega} = \frac{1}{N - v} - \delta_j$$

Where

$$\delta_j = \frac{N}{(N-\nu)^2} \sum\nolimits_{P/P(\nu)} \varepsilon_i + \frac{N^2}{(N-\nu)^3} \left(\sum\nolimits_{P/P(\nu)} \varepsilon_i\right)^2 + \varepsilon_j \frac{N}{N-\nu} - \varepsilon_j \left(\frac{N}{N-\nu}\right)^2 \sum\nolimits_{P/P(\nu)} \varepsilon_i$$

The expression (4A) for C_2 is a symmetric polynomial in δ_*, which are perturbations of the probability distribution. Therefore, the sum of all δ_* is equal to zero and the expression for C_2 does not depend on the linear approximation for δ_* and, consequently, on the linear approximation for ε_*. Denote the constant term in this expression by A. Thus in linear approximation,

$$T\left(P(\nu)\right) = C_1 C_2 = A((1-\frac{\nu}{N})^n - n(1-\frac{\nu}{N})^{n-1}(\sum\nolimits_{P(\nu)} \varepsilon_i)) \quad (7A)$$

From (7A) it follows that the quadratic approximation of the numerator in (2A) is:

$$\sum\nolimits_{P(\nu)} (\varepsilon_{i_1} + \varepsilon_{i_2} + \ldots + \varepsilon_{i_\upsilon}) T\left(P(\upsilon)\right) = \sum\nolimits_{P(\nu)} An(1-\frac{\nu}{N})^{n-1} \sum\nolimits_{P(\nu)} \varepsilon_i)^2 = -An\left(1-\frac{\nu}{N}\right)^{n-1} \left(\binom{N-1}{\nu-1} - \binom{N-2}{\nu-2}\right)(\sum\nolimits_p \varepsilon_i^2) \quad (8A)$$

The constant term in the denominator of (2A) is also obtained from (7A) by averaging over all the probability sets:

$$\sum\nolimits_{P(\nu)} T\left(P(\nu)\right) = \sum\nolimits_{P(\nu)} A((1-\frac{\nu}{N})^n = \binom{N}{\nu}(A\left(1-\frac{\nu}{N}\right)^n \quad (9A)$$

Thus from (8A) and (9A), taking into account the above-calculated constant term (1A), the sought for quadratic approximation of mathematical expectation of the probability sum for empty boxes is obtained:

$$\frac{\nu}{N} - \frac{An\left(1-\frac{\nu}{N}\right)^{n-1}\left(\sum\nolimits_p \varepsilon_i^2\right)}{\binom{N}{\nu} A\left(1-\frac{\nu}{N}\right)^n}\left(\binom{N-1}{\nu-1} - \binom{N-2}{\nu-2}\right) = \frac{\nu}{N} - \frac{\nu n}{N-1}(\sum\nolimits_p \varepsilon_i^2)$$

For occupied boxes, the above value must be subtracted from 1:

$$\frac{N-\nu}{N} + \frac{\nu n}{N-1}(\sum\nolimits_p \varepsilon_i^2)$$